Achievement in problem solving and metacognitive thinking strategies among undergraduate calculus students

PONNIAH, Logendra Stanley
Taylor's college, Malaysia

Abstract: The purpose of this study was to investigate problem solving in the field of calculus. The study developed and operationalized a metacognitive thinking strategies model. This model was then tested for its reliability and its predictive nature towards problem solving skills in non-routine calculus problems. A questionnaire was then administered among 480 first year undergraduate students who were selected randomly. The rate of return was about 90%. Using principal component analysis (PCA) the study successfully identified seven underlying dimensions of metacognitive thinking strategies. They are Self-efficacy, Define, Explore, Accommodate, Strategize, Execute and Verify. Finally, the researcher applied multiple regression analysis to evaluate the predictive ability of the identified predictor and the performance on routine and non-routine calculus problems. The study found that problem solving skills is acquired through practice and utilization of thinking strategies which is the corner stone on which advance mathematical ideas and particularly calculus are built on. This study revealed that there are six meaningfully predictive factors of calculus problem solving performance. It was found that "strategize" is the major predictive of calculus problem solving performance, followed by "accommodate, self-efficacy, define, explore and then execute". Further analysis revealed that Strategies, Accommodate and Self-Efficacy were considered most significant with substantial practical importance. With these findings, educators will be able to clinically evaluate a person's ability to regulate, monitor and control his or her own cognitive processes. Instructional strategies can then be developed for those individuals having difficulty functioning in the learning environment.

Keywords: Metacognition, problem solving, Mathematics Education.

The purpose of this study is to develop an instrument to adequately identify metacognitive strategies utilized by individuals' in the processes of solving mathematical problems. This study is particularly interested in measuring metacognitive strategies used by first year undergraduate students. It attempts to explore some potential correlation between the acquisition of metacognitive strategies and the actual mathematical problem solving skills. Within this study, the researcher would like to quantitatively validate the notion that metacognitive behavior does enhance the problem-solving ability. The principal objective of this study is to come up with a psychometrically sound self-reporting questionnaire that will measure the student's metacognitive awareness within the context of calculus. Simultaneously this study will make an attempt to reveal the predictive relationship between metacognitive behavior and the ability to solve mathematical problems.

The concept of metacognition

There are substantial research findings that substantiate the claim that metacognition enhances problem-solving behavior among its pupils. According to Davidson & Sternberg, (1998) "Metacognition appears to function as a vital element contributing to successful problem solving by allowing an individual to identify and work strategically. This link between metacognition and success in mathematical problem solving through an interplay between cognitive and metacognitive behaviors is well documented in the literature (Artzt & Armour-Thomas 1992; Carr and Biddlecomb, 1998; Linn 1987; Quinto & Weener, 1983).

Metacognition was introduced in the literature on metamemory by Flavell, Friedrichs and Hoyt (1970). Schoenfeld later introduced the notion of control as a medium instrumental in cognitive strategies meant for resource allocation (1992). Metacognition is an umbrella term for the knowledge and methodical regulation of such strategies. Similarly, Garofalo and Lester (1985) see control as part of metacognition. Metacognition has two separate but related aspects:

1. Knowledge and beliefs about cognitive phenomena,
2. The regulation and control of cognitive actions (p. 163).

Flavell (1971) definition of a metacognitive process as involving goal-directed, future-oriented mental behaviors that can be used to accomplish cognitive tasks clearly depicts the relationship between the two concepts. Flavell defined metacognition as, "knowledge and cognition about cognitive phenomena" (1971: p. 906). Flavell's idea of "knowledge and cognition about cognitive phenomena" is also germane to any attempt to shed light on the concept of metacognition. The idea of voluntary, goal-directed thinking applied to one's thoughts to realise cognitive tasks is deeply rooted in Piaget's conceptualization of formal operations. Its process of formal operations presupposes a hierarchy of thoughts in which the higher order thinking operates on the lower levels.

A cognitive process – whether it is automatic or not, conscious or unconscious- is certain to be either induced, prompted or encouraged by metacognitive knowledge in an enterprise of deliberate conscious memory. Accordingly, metacognitive knowledge gets quickly involved and automatically reconciles, adjusts and recognizes the expansive bank of metacognitive experiences. This is probably what Flavell often describes as the conscious cognitive or affective experiences that accompany our actions by ascribing to an intellectual enterprise. Thus, metacognition involves the "active monitoring and consequent regulation and orchestration" of cognitive processes in order to achieve cognitive goals (Flavell 1976:p. 252).

Kluwe (1982) elaborated on Flavell's theory of metacognition and shed new light on this empirical concept. He identified two other general attributes common to activities referred to as 'metacognitive'. Both attributes identified by Kluwe have to do with the person who exercises metacognitive thinking. The first postulates that “the thinking subject has some knowledge about his own thinking and that of other persons,” while the second suggests that “the thinking subject may monitor and regulate the course of his own thinking, i.e., may act as the causal agent of his own thinking" (1982, p. 202). Coincidentally, all processes seek to adapt and regulate the multitude of solutions actively.

Metacognitive thinking, presupposes self-awareness and promoting the metacognitive thinker as an actor in his or her environment besides being a deliberate reservoir and retriever of information. It seems reasonable, therefore, to adopt a convention that many researchers (e.g., Borkowski & Muthukrishna, 1992; Bracewell, 1983; Carr, Alex, Ander, & Folds-Bennett, 1994; Davidson, Deuser, & Sternberg, 1994; Paris & Winograd, 1990) have suggested which reserved the term metacognitive to the conscious and deliberate thoughts that have other thoughts as their object. Only when they are conscious and deliberate, are metacognitive thoughts potentially controllable by the person experiencing them, and only then, is the person potentially reportable and therefore accessible to the researcher. This convention will be adopted throughout the remainder of this chapter.

Method

The target population for the study comprised 1125 first year undergraduate students of the National University of Malaysia (*UKM*) who were either studying calculus at the time of the study or had just completed calculus courses recently. These students were mainly from the Faculty of Engineering, Faculty of Science and Technology, and the Faculty of Technology and Information Science. The sample of this study consisted of 480 undergraduate students drawn from a population of 1125 using systematic random sampling. The return rate was 90% (433 respondents), which was sufficient to address the study.

The PCA was employed to identify the underlying factors influencing students' metacognitive thinking behavior and problem solving skills in the field of Calculus.
Data on students' perceptions towards metacognitive thinking was collected based on 48 different items developed by the researcher. The inter-variable relationships were examined using Barlett's Test of Sphericity. This indicated that the inter-variable relationship was statistically significant (χ^2 (435) = 4402.452, p = 0.001); the Kaiser-Meyer-Olkin (KMO) measure of the overall sampling adequacy (MSA) was .903, which demonstrated the strong intercorrelation among the items. The individual MSA scores that indicated the intercorrelation within the items also ranged from .840 to .941 which suggested that the there is a high degree of correlation among the items and indicates the appropriateness of the use of factor analysis. The measures of commonality of items displayed a majority of scores of .50 and greater. The results of all the above statistical tests conducted suggested for the appropriateness of running PCA.

The result of the PCA extracted 30 components out of the given items (Appendix 1). Seven components with eigenvalues greater than one were retained for this study. These seven components explained around 59.53% of the cumulative variances in the data. The data was able to generate all the seven hypothesized factors. Hence, the researcher has identified these seven components as responsible for students' metacognition thinking behavior towards problem solving. These seven components were identified as self-efficacy, exploring, defining, accommodating, strategizing, executing and verifying respectively.

Accommodate

"Accommodate" was found to be the most significant factor in the student's perception towards metacognitive thinking behavior. It accounts for 28.49% of the overall variance. This factor was framed based on three guiding principles. The first is the students' ability to reorganize information within their schemata to meet the specific requirements of the question. This can be done by looking for specific terminology that may trigger certain algorithms. Finally, the students' ability to correlate what is given with what is known, judging from the given question. When the questionnaire was constructed, the researcher hypothesized five items to measure this factor. Based on the rotated factor analysis, all five items appear to be appropriate and hence all 5 items were retained.

Table 1 *Scores of Eigenvalues for each Factor*

Component	Initial Eigenvalues		
	Total	%of Variance	Cumulative %
Accommodate	8.55	28.49	28.49
Strategize	2.14	7.13	35.62
Explore	1.69	5.62	41.25
Define	1.54	5.13	46.38
Self-Efficacy	1.49	4.97	51.35
Execute	1.29	4.31	55.66
Verify	1.16	3.87	59.53

Extraction Method: Principal Component Analysis.

Strategizing

The second most important factor in this study was "Strategizing" which accounted for 7.13% of variation. This factor was framed based on the premise that once a student knows what to do; he or she proceeds to figure out how to do it efficiently. It involves the problem solvers quest, for the most efficient way to attain the desired result given a unique situation. At the initial stage of the questionnaire, six items were listed to measure this factor. Base on the rotated factor analysis, however, only five of them turned out to be appropriate and were, thus retained.

Exploring

The third most important factor was "Explore" which contributed 5.62% of variation explained. It is commonly perceived that one's ability to explore enhances his or her problem solving skills. These items, operationalize the skill of exploration, were constructed on the grounds of three assumptions, namely, (a) the students' ability to look back and review their past experience; (b) the respondents' ability to put together all the ideas and all its permutations; and (c) the students' ability to come to terms with all the possible situations that can be derived from the question. When the questionnaire was initially constructed, the researcher hypothesized 12 items to measure this factor. Nevertheless, in light of the rotated factor analysis, only nine items were retained.

Defining

Accounting for 5.13% of total variance explained, 'defining' is the fourth factor. The researcher designed 11 items that would successfully measure this construct. Based on the rotated factor analysis, only four items turned out to be appropriate and retainable. This factor was developed based on four criteria. The first criterion has to do with the students' ability to rephrase a given problem so that it matches their own schemata. The second pertains to the students' ability to reflect and match the domain specific vocabulary and definition to prescribed algorithm or mathematical models. The third principle is the student's ability to organize information meaningfully, whereas the final factor is the students' ability to mentally represent the given situation with the use of graphical aids.

Self-Efficacy

Self-Efficacy was found to be the fifth most significant factor that contributes towards student's perception of metacognitive thinking behavior. This factor accounted for 4.97% of the total variance explained. It was framed based on three guiding principles. The first is the students' perception that they find mathematics enjoyable. Positive feelings towards mathematics will indirectly pave the way to enhancing the students' perception of their own confidence level. Secondly, self-efficacy has a correlation to their past success. When the questionnaire was constructed, the researcher hypothesized three items to measure this factor. The rotated factor analysis confirmed all three items as appropriate and retainable.

Execute

The sixth factor is execution. The items for this factor was constructed based on two premises, namely (a) the students' work through the problem until the end using a prescribed algorithm, and (b) the assumption that students will not make and attempt to presume the final outcome and cut short their endeavor. At the construction, stage the researcher hypothesized six items to measure this factor. All six items were confirmed by the rotated factor analysis and thus retained.

Verification

The seventh and the last factor is verification. The factor verification built on the assumption that successful problem solvers tend to monitor their solutions. In other words, successful students have the inclination to verify that their outcome would match their interpretation of the demand of the question. At the design, stage the researcher conjectured 5 items to gauge this behavior. After administering factor analysis, it was found that only three items were appropriate to be retained. Hence, these seven factors (Appendix 2) were extracted for further analysis to answer the research questions.

Predictors of Metacognitive Thinking Strategies

The first regression analysis examined the predictors of metacognitive thinking strategies. Six predictors (Self-Efficacy, Explore, Define, Accommodate, Strategize, and Execute) were entered into the multiple regression analysis of the Statistical Package for the Social Sciences (SPSS) program in order to determine the strength and predictive ability of the seven independent variables in explaining variations in students' use of Metacognitive thinking strategies in their studies.

Table 2 summarizes the descriptive statistics (mean scores and standard deviations) and correlations among the predictors and the criterion variable (metacognitive thinking) as yielded by the SPSS computer out-puts. The analysis of the correlations revealed that three of the six predictors were statistically significant. Accommodate, Strategize and Self-Efficacy was statistically significant.

Table 3 summarizes the results of the regression analysis on the Metacognitive Thinking Strategies. In this model, students' reported use of thinking strategies, the criterion variable was tested using six predictors, namely Self-Efficacy, Explore, Define, Accommodate, Strategize, and Execute. Analysis of Variance (ANOVA) revealed that the overall model was statistically significant; $F(6, 195) = 46.207$, $\underline{p} = 0.001$, MSE = 146.371, and the set of the independent variables accounted for 59% of the total variance explained. The adjusted coefficient of determination (adjusted R^2) was .57, with an estimated standard error of 12.09. Further analysis of the predictive power of the individual predictors indicated that all the factors in the study were statistically significant.

The estimated equation model can be summarized as follows:
$\hat{Y} = 38.79 + 8.38 (S) + 7.89 (A) + 7.42 (G) + 2.68 (D) + 2.39 (X) + 2.33 (E)$.

Table 2 *Inter-Variable Correlations and Descriptive Statistics for the Predictors and Performance*

	Achievement Score	Accommodate	Strategize	Explore	Define	Self-Efficacy	Execute	Verify
Achievement Score	1.00	.46	.43	.08	.11	.39	.08	-.01
Accommodate	.46	1.00	.05	-.01	.03	.03	-.12	-.03
Strategize	.43	.05	1.00	-.04	-.11	-.03	-.04	-.01
Explore,	.08	-.01	-.04	1.00	.01	-.06	-.02	-.07
Define,	.11	.03	-.11	.01	1.00	-.01	.01	.08
Self-Efficacy	.39	.03	-.03	-.06	-.01	1.00	.02	-.02
Execute	.08	-.12	-.04	-.02	.01	.02	1.00	.03
Verify	-.01	-.03	-.01	-.07	.08	-.02	.03	1.00
Mean	37.78	-.02	-.02	-.05	.02	-.06	-.08	.12
Std. Deviation	18.54	1.03	.99	1.03	1.01	1.00	1.09	.95

Table 3 *Summary Statistics of the Regression Coefficients Confidence Intervals, Collinearity Statistics, and Threshold of Practical Importance for the Predictors of Metacognitive Thinking Strategies*

Model						95% Confidence Interval		Collinearity Statistics		
	B	Std. Error	Beta	t	p	Lower Bound	Upper Bound	Tolerance	VIF	Threshold
(Constant)	38.79	.86		45.25	.01	37.10	40.48			
Accommodate(A)	7.89	.84	.44	9.42	.01	6.24	9.54	.98	1.02	1.80
Strategize(S)	8.38	.87	.45	9.61	.01	6.66	10.10	.98	1.02	1.88
Explore(E)	2.33	.83	.13	2.79	.01	.68	3.97	.99	1.01	1.81
Define(D)	2.68	.85	.15	3.16	.01	1.01	4.35	.99	1.01	1.83
Self-Efficacy(G)	7.43	.86	.40	8.66	.01	5.73	9.12	.99	1.01	1.86
Execute(X)	2.39	.79	.14	3.04	.01	.84	3.94	.98	1.02	1.70
Verify(V)	.23	.91	.01	.25	.80	-1.564	2.03	.99	1.01	1.96

Dependent Variable: achievement score
(Alpha is significant at $p < 0.05$)

Findings

When it came to authentic problem solving in calculus, experts generally use a top down approach whereby they initially perform qualitative analysis to identify the applicable principles, concepts, and procedures. Only then can they attend to the details by applying the procedures to generate a solution to the problem at hand. In contrast, novices often employ a means-ends analysis, which consists of attempting to reduce the distance between a problem's initial sate and the goal state.

The results from the principal components provided some clarification of the construct as conceptualized by the researcher. A major purpose of the current study was to assess the validity of the components underlying the metacognition construct and to ascertain the nature of the relationships among them. The results lent some empirical support for the notion that metacognition is a multidimensional construct, the components of which work in interaction. These seven components were able to explain around 60% of the total variance. Among these the higher variances were explained by Accommodate, Strategize, Explore and Define respectively, which constituted around 50% of the total variance. Desoete and Roeyers (2002) also found metacognition to be multidimensional construct; they further added that this will enable learners to adjust to varying tasks, demands and contexts. At the same time Boekaerts (1999) cautions us about metacognition, which is often used in an over, inclusive way, including motivational and affective constructs.

Metacognition thinking strategies is essential for any extended activity, especially problem solving, because the problem solver needs to be aware of the current activity and of the overall goal, the strategies used to attain that goal, and the effectiveness of those strategies. It is well established that successful students possess powerful strategies for dealing with novel problems, can reflect on their problem-solving actions, and can monitor and regulate those strategies efficiently and effectively (Campione,1989; English, 1992; Flavell, 1979; Lawson, 1990; Peterson, 1988). In her naturalistic studies, Peterson found students' abilities to diagnose and monitor their own understanding to be a significant predictor of their mathematics achievement. Students who were able to provide a good explanation of which particular mathematics problem or lesson component they were unable to understand and why, tended to have significantly higher scores on a test of mathematics achievement.

This study was able to authenticate this result. The multiple regression analysis carried out on the collected data based on the seven components showed a significant relationship between the students' performance with these seven components. This indicates that these components strongly contribute towards the achievement scores by students, which ultimately shows the relationship with problem solving skills in calculus of students.

The analysis showed that the Strategize component was the most significant predictor of the six significant components. This finding is consistent with what Colbeck, Campbell and Bjorklund (2000) found in their study, they found that successful students were more able to determine a feasible and cost-effective solution, to either build or model their solutions. Fractional concepts, ideas, or algorithms must be organized and built on the learner's existing knowledge. According to Halpern (1996) weak students have the tendency to use "trial-and-error approach in selecting a strategy, which in itself is a poor strategy. The researcher settled on measuring this component by measuring the student's ability to find the most efficient way to attain the result.

The second most significant predictor was Self-efficacy. Various researchers such as Hackett, (1985); Pajares, (1996); Pajares & Miller, (1994) have reported that students' judgments of their capability to solve mathematics problems are predictive of their actual capability to solve novel and non-routine problems. Self-efficacy in mathematics also has been shown to be a strong predictor of mathematical problem-solving capability (Pajares & Kranzler, 1995). Most importantly researchers such as Collins, (1982) and Schunk, (1989, 1991) have reported that, when students approach academic tasks, those with higher self-efficacy work harder and for longer periods of time than do those with lower self-efficacy. Students who experienced more enjoyment while learning mathematics achieved higher scores. Beliefs also have been shown to have an influence on achievement (Garofalo, 1989; Kloosterman, 1995; Schoenfeld, 1985). However, Young and Ley (2002) remind us that self-efficacy is not the only influence on achievement behavior. High self-efficacy will not produce competent performance when requisite knowledge and skills are lacking.

Accommodate was the third most significant predictor. This is probably one of the most crucial qualitative or subjective stages of solving a non-routine or a novel problem. This is where the student puts together his interpretation of the question and his knowledge about the subject domain, in this case in calculus. At this point the student has completed the process of understanding the question and its connection to the subject domain. At this stage of the operation, the student knows exactly what the question is, what one can possibly do with the information provided within the question and most importantly, the student has come to a realization as to what he or she needs to find. The student reconstructed fractional ideas from the questions and from his memory into an organized and structured entity. This is where Reimann & Schult, (1996) calls for a "plan, a very specific solution plan that contains no generalized conditions and actions, but only specific ones". Miller and Pajares (1997) further add that students with substantial experience in problem solving will use this stage to predetermine the amount of time and effort they put into solving those problems. This study found the three components:

The fourth significant metacognitive predictor was Define. This component measured the student's abilities to define the question and put it in some form of operational context. In this case, for a student to be able to define a problem, he or she must be able to restructure the problem in his or her own words that matches the student's domain specific schemata. The student must be capable of forming a mental representation or graphical representation of the question at hand. The student must also be able to understand the actual meaning of the specific words, which in this case were the mathematical definitions. According to Owen and Fuchs (2002) the ability to define a problem is to understand the problem from the problem solvers own perspective. This stage of problem solving is functionally significant for the problem solver. The perception of the problem may change during the course of problem solving. It may shrink or expand according to the actions and constructions developed during solution. Furthermore, the ability to define a problem is crucial to problem comprehension, which may prevent surface level analysis.

The fifth predictor for achievement is the ability to explore a problem. At this stage the students go through some sort of a brain storming session. The student is required to qualitatively analyze the problem. According to Merseth, (1993) this stage is not only important in solving the mathematical problem but also to pave the way for further deeper understanding within a particular subject domain. Rickard (1995) echoed this sentiment and further added that exploring problems will build mathematical connections. To explore the problem successfully the students have to go through all the different permutation between the known and the unknown. This is demanding and it challenges the student's ability to reach deep down into his or her own knowledge reservoir and come up with all the possibilities.

The sixth and final significant predictor was the ability to execute. This is the actual working out of the algorithms. A process-constrained task requires students to carry out a procedure or a set of routine procedures in solving the problem. Philipp, (1996) suggested that people tend to streamline the process by which we compute invent computational algorithms. However, what is important is that the student should in most cases work the question till the end and not resort to guessing or anticipating the solution.

What is surprising is that the research result indicated that the Verify predictor was statistically insignificant. This component was intended to measure the student's ability to monitor his or her own strategy or work. Solution monitoring allows an individual to analyze the problem requirements, deliberate construction and evaluation of problem representations and the effectiveness of their procedures. This component includes an individual's control over the internal representations he or she has formed and still needs to form for understanding and solving a problem. Often, new strategies need to be formulated as a person realizes that the old ones are not working.

The results of the current study, although clarifying some important issues regarding the construct of metacognition, are by no means definitive. It has to be admitted that the validation of a construct, particularly one as elusive as metacognition, is a lengthy process; the present investigation represents only a step in this direction

Mathematics lecturers must undertake more responsibility in their students thinking processes. We as lecturers must create an equilibrium between teaching subject matter and at the same time molding the cognitive construct of the students. With the development of information and communication technology, information has lost its premium. With a click of a button we have access to boundless information, facts and figures. This calls for a paradigm shift. Educators should no longer be too concerned with hunting and gathering information, but the emphasis now should be more on how to synthesize and make sense of information.

The metacognitive thinking strategies modeled in this study did prove that it had a significant predictability. The results of the study successfully demonstrated that successful problem solvers think in a structured manner and this study to certain degree was able to quantify this structure. The question is how, can we exploit this finding. The suggestion is that lecturers must act as models of problem solvers in the class. Lecturers and teachers have always pre-solved problems before they demonstrate the solution in class. This is probably what created a false impression that solutions must be attained in the first attempt. Lecturers must be bold enough to paint the real picture, where they only arrive at the final solution after numerous false starts. The lecturers should also think aloud in class to expose to the students their thinking processes. The students must be brought to be aware that lecturers are actually systematically working out the problem rather then just reproducing the answer from memory.

Lecturers should prepare some sort of checklist for the students at the beginning of the lesson. Students should be monitored and rewarded for organizing their thoughts in congruence with this checklist. This checklist should include components of the metacognitive thinking strategies that have been discussed above. This researcher is certain if this is done at an early stage and repeatedly until their execution becomes a natural behavior (automatic habit).

This study was able to demonstrate that self-efficacy is one of the predominant factors in the students' problem solving achievement. Lecturers and particularly teachers should take heed of this discovery. Students should be psychologically prepared particularly at the younger age, to escalate their efficacy for mathematics and in particular problem solving. Teachers and lecturers should be encouraged to allocate significant amount of their time to promote the desire for acquiring their subject matter. This will tantalize the students' interest and at the same time motivate the students to acquire these information and skills voluntarily.

Mathematics lecturers must participate in building up the cognitive construct from the inside out. Lecturers must embrace the view that they are responsible to bring the students to mathematics rather then to bring mathematics down to the students. In other words, students should be prepared to participate in this infinite journey of mathematics rather than just diluting the ideas and its functionality to barely meet surface level current demands.

References

Artzt, A. F., & Armour-Thomas, E. (1992). Development of a Cognitive-Metacognitive Framework for Protocol Analysis of Mathematical Problem Solving in Small Groups. *Cognition and Instruction, 9*(2), 137-175.

Boekaerts, M. (1999). Metacognitive experiences and motivational state as aspects of self-awareness: Review and discussion. *European Journal of Psychology of Education*, 14, 571-584

Borkowski, J. G., & Muthukrishna, N. (1992). Moving metacognition into the classroom: "Working models" and effective strategy teaching. In M. Pressley, K. R. Harris, & J. T. Guthrie (Eds.), *Promoting academic competence and literacy in school* (pp. 477-501). San Diego, CA: Academic.

Bracewell, R. (1983). Investigating the control of writing skills. In P Mosenthal, L. Tamor, & S. A. Walmsley (Eds.), *Research on writing: Principles and methods* (pp. 177-203). New York: Longman.

Campion, J. C. (1989). Assisted assessment: A taxonomy of approaches and an outline of strengths and weaknesses. *Journal of Learning Disabilities, 22* (3), 55-65.

Carr M., Alexander J., & Folds-Bennett T. (1994). Metacognition and mathematics strategy use. *Applied Cognitive Psychology*, 8, 583-595.

Carr, M., & Biddlecomb, B. (1998). Metacognition in mathematics: From a constructivist perspective. In D. J. Hacker, J. Dunlosky, & A. C. Graesser (Eds.), Metacognition in educational theory and practice (pp. 69-91). Mahwah, NJ: Lawrence Erlbaum Associates.

Colbeck, C. L., Campbell, S. E., & Bjorklund, S. A. (2000). Grouping in the Dark. *Journal of Higher Education, 71*(1), 60.

Davidson J. E., Deuser R., & Sternberg R. J. (1994). The role of metacognition in problem solving. In J. Metcalfe & A. R. Shimamura (Eds.), *Metacognition: Knowing about knowing* (pp. 207-226). Cambridge, MA: MIT Press.

Davidson, J. E., & Sternberg, R. J. (1998). Smart problem solving: How metacognition helps. In D. J. Hacker, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in educational theory and practice* (pp. 47-68). Mahweh, NJ: Lawrence Erlbaum Associates.

Desoete, A., & Roeyers, H. (2002). Off-Line Metacognition-A Domain-Specific Retardation in Young Children with Learning Disabilities? *Learning Disability Quarterly, 25*(2), 123-172.

English, L. D. (1992). Problem solving with combinations. *Arithmetic Teacher, 40*(2), 72-77.

Flavell, J. H. (1971). First discussant's comments: What is memory development the development of? *Human Development, 14*, 272-278.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. Resnick (Ed.), *The nature of intelligence.* Hillsdale, NJ: Erlbaum.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive developmental inquiry. *American Psychologist, 34*(10), 906-911.

Flavell, J. H., Friedrichs, A.G., & Hoyt, J.D. (1970). Developmental changes in memorization processes. *Cognitive Psychology*, *1*, 323–340.

Garofalo, J. (1989). Beliefs and their influence on mathematical performance. *Mathematics Teacher, 82*, 502-505.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176.

Hackett, G. (1985). The role of mathematics self-efficacy in the choice of math-related majors of college women and men: A path analysis. *Journal of Counseling Psychology, 32*, 47-56.

Halpern, D. F. (1996). *Thought and Knowledge An Introduction to Critical Thinking* (3rd edn.). Mahwah, NJ: Lawrence Erlbaum Associates.

Kloosterman, P. (1995). *Students' Beliefs about Knowing and Learning Mathematics: Implications for Motivation*. Cresskill, N.J.: Hampton Press.

Kluwe R. H. (1982). Cognitive knowledge and executive control: Metacognition. In D. R. Griffin (Ed.), Animal mind - human mind (pp. 201-224). New York: Springer-Verlag.

Lawson, H. A. (1990). Beyond positivism: Research, practice, and undergraduate professional education. *Quest, 42*, 161-183.

Linn, M. M. (1987). *Effects of journal writing on thinking skills of high school geometry students*. University of Northern Florida: Masters of Education Project.

Merseth, K. K. (1993). How Old Is the Shepherd? An Essay about Mathematics Education. *Phi Delta Kappan, 74*(7), 548-592

Miller, M. D., & Pajares, F. (1997). Mathematics Self-Efficacy and Mathematical Problem Solving: Implications of Using Different Forms of Assessment. *Journal of Experimental Education, 65*(3), 213-228.

Owen, R. L., & Fuchs, L. S. (2002). Mathematical Problem-Solving Strategy Instruction for Third-Grade Students with Learning Disabilities. *Remedial and Special Education, 23*(5), 268-281.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research, 66*, 543-578

Pajares, F., & Kranzler, J. (1995). Self-efficacy beliefs and general mental ability in mathematical problem-solving. *Contemporary Educational Psychology, 20*, 426-443.

Pajares, F., & Miller, M. D. (1995). Mathematics Self-efficacy and mathematical performances: The need for specificity of assessment. *Journal of Counseling Psychology, 42*, 190-198.

Paris S. G., & Winograd P. (1990). How metacognition can promote academic learning and instruction. In B. E Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp. 15-51). Hillsdale, NJ: Lawrence Erlbaum Associates.

Peterson, P. L. (1988). Teachers' and students' cognitional knowledge for classroom teaching and learning. *Educational Researcher, 17*(5), 5-14.

Philipp, R. A. (1996, November). Multicultural Mathematics and Alternative Algorithms. *Teaching Children Mathematics*, *3*, 128-130.

Reimann, P., & Schult, T. J. (1996). Turning Examples into Cases: Acquiring Knowledge Structures for Analogical Problem Solving. *Educational Psychologist, 31*(2), 123-132.

Rickard, A. (1995). Teaching with Problem-Oriented Curricula: a Case Study of Middle-School Mathematics Instruction. *Journal of Experimental Education, 64*(1), 3-26.

Schoenfeld, A. H. (1985a). *Mathematical problem-solving.* Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem-solving, metacognition, and sense-making in mathematics. In D. Grows (Ed.), *Handbook on research on mathematics teaching & learning.* New York: Macmillan Publishing Company.

Schunk, D. H. (1989). "Self-efficacy and achievement behaviors". *Educational Psychology Review*, 1, 173-208.

Schunk, D. H. (1991). "Self-efficacy and academic motivation". *Educational Psychologist*, 26, 207-231.

Young, D. B., & Ley, K. (2002). Brief Report: Self-Efficacy of Developmental College Students. *Journal of College Reading and Learning, 33*(1), 21-52

Appendix 1

Inter-Variable Correlations and Descriptive Statistics for the Predictors and Metacognitive Thinking Strategies

(Alpha is significant at $p<0.05$)

	G1	G2	G3	D4	D5	D6	D7	E15	E16	E21	E22	E23	E25	A27	A28	A29	A30	A31	S32	S33
G1	.860																			
G2	-.540	.884																		
G3	-.269	-.137	.924																	
D4	.025	-.010	-.122	.902																
D5	-.030	-.026	-.017	-.341	.890															
D6	.054	.016	-.070	.026	-.147	.872														
D7	.021	-.039	.077	-.117	-.189	-.444	.865													
E15	-.022	-.023	.007	-.100	-.047	-.011	.008	.887												
E16	.023	-.036	.000	-.082	.038	.009	-.104	-.290	.877											
E21	-.078	.009	.106	.054	.042	-.072	-.008	.060	-.100	.929										
E22	.040	.002	-.008	-.080	.029	-.039	.007	-.031	-.018	-.194	.915									
E23	.007	.002	-.069	.127	-.057	.039	-.049	-.102	-.197	-.146	-.252	.899								
E25	-.060	-.016	-.029	.014	-.019	-.069	.022	-.093	-.214	-.091	-.137	-.053	.919							
A27	.030	-.035	-.062	-.005	.010	-.019	-.058	.082	.049	-.038	-.100	.060	-.086	.938						
A28	-.024	-.066	-.007	-.131	.072	-.037	.021	-.053	.047	.021	-.057	-.074	-.033	-.117	.941					
A29	-.039	-.047	-.039	-.043	-.014	.067	.025	-.018	.030	-.093	-.071	.037	.061	-.137	-.214	.937				
A30	-.049	-.044	.061	.039	-.038	-.136	.030	.054	.021	.020	-.032	.060	-.152	.025	-.057	-.201	.934			
A31	-.083	.046	.002	-.015	-.035	-.067	-.018	-.002	.001	.024	.077	-.106	.030	-.231	-.186	-.183	-.093	.928		
S32	-.001	-.054	-.019	-.101	.079	-.041	.017	.025	.064	-.011	-.105	-.002	-.012	.011	.114	-.056	.039	-.077	.922	
S33	-.059	-.050	-.064	-.054	-.046	.009	.062	.021	.031	-.094	.102	.012	-.033	-.092	-.063	-.044	-.100	.037	-.201	.941
S34	-.002	.011	-.030	-.025	.039	.047	-.067	-.003	-.034	-.009	.053	-.047	-.113	-.031	-.067	.004	-.051	-.051	-.262	-.229
S35	-.097	.064	.051	-.060	.029	-.078	-.069	-.069	-.025	-.046	-.014	-.009	.073	.054	-.039	-.061	.121	.010	-.095	-.096
S36	.092	-.042	-.031	.018	-.064	.044	.030	-.024	-.037	.014	-.035	-.087	.007	-.017	-.048	.021	-.097	-.019	-.119	-.137
X38	-.037	.002	-.034	.047	-.011	-.026	.059	-.028	-.073	.035	-.011	.029	.078	-.140	-.069	.002	-.038	.058	-.169	.034
X40	.035	-.042	.020	-.019	-.029	.052	-.035	.004	.033	-.081	-.039	.042	-.085	.066	-.003	.062	.034	-.136	-.028	-.071
X42	.002	.037	-.064	.049	-.051	.013	-.125	-.187	-.027	-.050	-.144	.096	.151	-.040	.023	.032	-.072	-.004	-.023	-.008
X43	.046	-.047	.066	-.010	-.012	-.029	-.002	-.006	.070	-.021	-.027	-.081	-.049	-.073	.044	.007	-.045	-.035	.051	-.028
V45	-.078	.084	.045	-.060	-.040	-.025	-.063	.057	-.093	-.016	.007	-.036	.004	-.051	-.030	-.009	-.030	.041	-.034	.023
V46	-.006	.022	-.032	.010	.093	.056	-.015	.022	-.062	-.008	.047	-.011	-.139	.058	.070	.002	-.085	-.059	-.042	-.061
V47	.048	-.073	-.046	.046	-.092	-.078	.012	.018	.053	-.005	-.003	-.031	.052	-.006	-.037	-.035	-.035	.083	.071	.049
Mean	3.42	3.42	3.09	3.34	3.47	3.88	3.82	3.44	3.35	3.55	3.65	3.49	3.32	3.54	3.47	3.51	3.32	3.58	3.29	3.31
SD	.887	.830	.874	1.069	.968	.947	.915	.983	.898	.854	.888	.926	.858	.933	.929	.902	.869	.955	1.007	1.011

Appendix 2

Loadings for the Seven-Factor Rotated Solution for the Metacognitive Thinking Strategies Dimensions

Item no.	Statement	Component						
		1	2	3	4	5	6	7
1	I feel comfortable with mathematics.					.788		
2	I enjoy working with mathematics.					.766		
3	I consider myself as a successful student in mathematics.					.728		
4	Before answering Q3 I ask myself is there sufficient information to compute the unknown?.				.600			
5	Before starting a problem I ask myself is there any contradictory condition in the problem?				.744			
6	Before starting a problems like Q2 or Q3 I always try to restate the problem in a way I can understand.				.683			
7	Before starting a problem like Q2 or Q3 I ask myself whether I have all the necessary data.				.751			
15	I try to remember whether I have seen something similar to this in a different context? (e.g. Q2 & Q3 involves a combination of distance between 2 points and minimum distance)			.559				
16	I ask myself 'Do I know how the problems are related? (e.g. Q2 & Q3 are almost the same question)			.720				
21	Whenever I am stuck on a problem, I ask myself, " How might a different strategy help me solve this problem?" (e.g. using coordinate geometry to solve this problem			.552				
22	When working on a problem I ask myself, "How might the information that I have learned in the past help me solve this problem?"			.591				
23	Before attempting to solve a problem I ask myself, "How is this problem like the ones that I have solved in the past?"			.704				
25	Before attempting to solve a problem I try to figure out how to put the different pieces of information together. (e.g. Q3 can be solved by using information from Q1& Q2			.579				
27	I determine what information in the problem is most relevant. (e.g. the escape rout of Waja is the tangent of the curve at (I,I))	.665						
28	I examine the question carefully looking for clues. (e.g. Q1 , Q2 & Q3 are related)	.627						
29	I look for key words or phrase that will help me solve the problem(e.g. the word maximum in Q1 and the word closest in Q3)	.688						
30	I always examine all aspects of each question before beginning to answer	.560						
31	While I am solving a problem I ask myself, "Am I using the appropriate rule or formula?"	.657						
32	I select strategies to carry out my plan		.674					
33	I identify major goals.(e.g. in Q1 first form the equation and then differentiate for finding the maximum.)		.618					
34	During attempting to solve the problem I always make sure that my steps are in the correct order.		.610					
35	I experience difficulty in solving a problem when I am unable to decide on a strategy (e.g. I don't know how to solve Q3 using calculus)		.737					
36	While attempting to solve a problem I try to think of a strategy that might help me to solve the problem.		.697					
38	I evaluate my strategies as I proceed						.641	
40	I do carry out all the necessary calculation before arriving at the						.563	
42	While solving a problem, I ask myself, "Am I leading in the right direction or am I on the right track?"						.747	
43	When I am stuck on a problem I ask myself, "Did I consider all the relevant information in the question?"						.774	
45	After I finished solving Q1, Q2 & Q3 I check to see if my answer make sense.							.802
46	After I have finished solving a problem I check to see if I used the right strategy. (e.g. for Q1 I combined Q1a &Q1b and differentiated them)							.783
47	After I have finished solving a problem I check to see if my answer corresponds to the question asked.							.776