Development of Mathematics Pedagogical Content Knowledge in Pre-service Teachers

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Abstract: The objective of mathematics methodology courses in teacher preparation programmes is concerned about the development of MPCK in their pre-service teachers. As part of a research study on the development of MPCK in primary school beginning teachers, a 16-item instrument was developed to measure some aspects of the MPCK for teaching mathematics at primary level. The instrument was administered to the 2005 Intake of the Postgraduate Diploma in Education (Primary) student teachers at the National Institute of Education, Singapore just at the beginning of their programme. As they complete their methodology course in February 2006, the instrument was administered again. This paper discussed the findings concerning their performance in these two tests, with reference to the overall performance as well as topic-specific and MPCK construct-specific performance. The findings indicate that student teachers at the beginning of their programmes are generally quite weak in their mathematics pedagogical content knowledge, as might be expected. There was significant improvement in most aspects of their MPCK on completion of their mathematics pedagogy course.
Keywords: mathematics pedagogical content knowledge, Postgraduate Diploma in Education (Primary) programme, subject knowledge

Introduction

Mathematics pedagogical content knowledge (MPCK) of teachers cannot be easily defined but is a complex concept integrating generic pedagogical knowledge, mathematics teaching methodology as well as knowledge of the discipline of mathematics. While some aspects of these could be innate, most of this knowledge is acquired through the teachers' own education comprising general education, mathematics education, professional teacher preparation and development courses, interacting with experiential learning on the job. The state of teachers' MPCK and the development of MPCK in teachers is thus worthy of study by mathematics teacher educators and should inform and have impact on the design of pre-service teacher education curriculum as well as professional development courses. From the viewpoint of teachers and school leaders, such research findings will contribute data towards needs analysis and hence influence choices of professional development courses while from the policy making angle, these findings can better inform policy decision processes.

Having the sole responsibility for pre-service teacher education, mathematics teacher educators at the National Institute of Education (NIE) in Singapore have always been concerned over the effectiveness of their methodology courses in developing MPCK in their student teachers. While course evaluation and feedback from student teachers and surveys of school administrators have been generally positive, such data is largely based on perceptions and it is substantially more difficult to collect data on student teachers' actual performance on mathematics teaching after they have graduated from the course. It is with this background that the research project entitled Knowledge for Teaching Primary Mathematics (or MPCK Project) was initiated in 2003 with the objective of studying the development of beginning primary school teachers' MPCK. The findings of the project will be able to provide data and information for the review of the mathematics methodology courses or of programme structures at the NIE. One of the research areas of this project is to measure the development of MPCK of the student teachers during their pre-service programme. At the beginning of the project, an instrument was developed to measure several aspects of teachers' MPCK. A detailed description of the instrument used in this study and its coding procedures can be found in Lim-Teo (2006). This instrument was administered to student teachers of the Postgraduate Diploma in Education (Primary) programme (or PGDE (Primary) programme) on their entering the programme and again towards the end of their programme. This paper describes their performance in the pre-test and the post-test with regards to the different aspects of their MPCK and also as pertaining to different mathematical topics.

Postgraduate Diploma in Education (Primary) programme

The Postgraduate Diploma in Education (Primary) programme (or PGDE (Primary) programme) is a one-year (or 2-semester) initial teacher preparation programme for primary school teachers. Although this programme has the academically able student teachers among the three primary teacher preparation programmes at NIE, it was important to ascertain their development of MPCK. In this programme, the student teachers are prepared to teach three subjects, namely, English, Mathematics and Science. A small number of them may replace Science with Social Studies, Art or Music. The basic level of Mathematics required before they can be admitted to the programme is a C6 grade or better in Elementary Mathematics at the GCE Ordinary Level examinations[1]. Other than this requirement for programme admission, all the student teachers in the programme are required to be prepared for mathematics teaching regardless of their background in mathematics. The general education qualification of the student teachers at point of entry into the programme is a degree from the National University of Singapore, the Nanyang Technological University or other universities whose degrees are acceptable to the education service of Singapore.

During the programme, student teachers undergo a methodology course for teaching mathematics. As this group of student teachers had a few months of contract teaching experience in the school, some of them were identified by school principals to undergo subject knowledge course to supplement their mathematical knowledge. The methodology course consists of two modules, one in each semester, with 72 hours of contact time in the first semester and 24 hours in the second semester. Course content includes Singapore's primary Mathematics curriculum, learning theories and learning difficulties, teaching strategies for a range of mathematics topics (whole numbers, fractions, decimals, space, graphs, ratio, rates, percentage, etc.) including the use of information technology in the teaching of these topics, problem solving through various heuristics, mathematical investigation, use of assessment tools and error analysis. The courses will not be too different from mathematics methodology courses in the United States and in fact American textbooks are used as references in these courses. Only one additional localized topic which involves the use of the "model method" in solving arithmetic problems, a method that primary mathematics teachers in Singapore schools must know and be able to teach. These pedagogy modules are also characterised by discussion; experimentation and presentation of teaching ideas. In addition, assessment of these courses include presentations, assignments and group projects involving lesson planning, error analysis, demonstration teaching, mathematical investigation, problem-solving and use of technology tasks.

The mathematics subject knowledge course consists of 2 modules of 24 hours each, to be

[1] The GCE O Level Examination is a national examination taken at age 16 with grades A1, A2, B3, B4, C5 and C6 being considered O level pass grades.

taken in the first semester of their programme. This course provides a deeper understanding of the mathematics content knowledge which is linked to the primary mathematics curriculum. It consists of topics such as historical numeration systems with special emphasis on place value and base, properties of operations on numbers, basic ideas of divisibility, proportional reasoning, problem solving with links to algebra, geometrical concepts and properties pertaining to triangles and quadrilaterals, measurement of space concepts and basic statistical concepts such as data representation and measures of central tendency and spread. The content course is assessed by tests and mathematics assignments.

During the programme, the student teachers has the opportunity to link theory to practice during one stint of teaching practice in schools known as practicum. In the programme when the research was conducted, the ten weeks practicum took place in the second half of the second semester, right at the end of the programme. During their practicum, the student teachers were supervised by mentor teachers in schools and overseen by faculty from the university. However, due to resource constraints, the faculty assigned to these student teachers were not from the mathematics education department but from general education departments. Thus faculty members who teach the mathematics pedagogy and mathematics courses are only able to see their students in action for research purposes rather than as part of their teaching duties.

Research Method

The instrument was administered to this sample of student teachers entering the PGDE (Primary) programme in July 2005. The instrument was again administered in February 2006, just before the student teachers embarked on their final teaching practice stint. During the period from July 2005 to February 2006, the development of their MPCK are effected by up to 4 modules (96 hours) of mathematics-related courses as described above and the generic education courses (another 96 hours). The whole cohort of the PGDE(Primary) July 2005 intake consisted of nearly 261 student teachers at the beginning of the programme but, as there were a few who left the programme. A sample of 113 student teachers was selected from this cohort to take both the pre- and post- tests.

The student teachers took the tests without any preparation in terms of studying or revision. Although no time limit was imposed, almost all finished the test in one hour. The pre-test was conducted during their first tutorial class while the post-test was conducted at the last tutorial class of the methodology course.

Findings and Discussion

Coding the items on the test scripts was carried out by three researchers. Firstly, the items were assigned the nominal codes independently by two researchers. A third researcher checked the coding and also resolved the few cases where there were differences in codes given by the first two coders. Krippendorff's α defined in Krippendorff (2004, pp 222-228) was used as a measure of the inter-coder reliability between the first two researchers. The value of α was calculated for each item or sub-item, and ranged between 0.62 and 1. The mean value of α was 0.85.

Before performing statistical analysis, the score of each item was scaled to a range from 0 to 4. The maximum score possible for each topic and each construct was thus 16, and the maximum total score possible was 64. As a preliminary analysis, paired t-test was performed one-at-a-time to compare the pre-test and post-test total scores as well as the individual scores for each item. The alternative hypothesis of each paired t-test was post-test mean > pre-test mean.

After completing 96 hours of mathematics pedagogy, there was a significant improvement (at the individual level) in the total scores. One-at-a-time paired t-tests in Table 1 suggested improvements (which might not be simultaneously significant) in all items except items 3, 6 and 8. It is interesting that item 6 was an anomaly as its post-test mean was slightly (but not significantly) less than its pre-test mean. Although a "content" item, the student teachers were not tested on their content knowledge per se but rather on their ability to represent three forms of answers in different contexts. The item required them to write one story problem for three answers. The answers were derived from 23 divided by 4 (23 ÷ 4) which were based on three different possible story contexts that showed three different possible answers.

Table 1: *Paired t-tests by item*

Item No.	Pre-test mean	Post-test mean	Difference of means	T	p-value (post > pre)
1*	2.27	3.08	0.81	4.61	< 0.001
2*	1.53	2.31	0.78	4.63	< 0.001
3	2.24	2.33	0.09	0.74	0.23
4*	1.33	1.81	0.47	3.27	0.001
5*	1.24	1.69	0.45	3.66	< 0.001
6	2.62	2.59	-0.03	-0.18	0.57
7*	2.98	3.22	0.24	2.00	0.024
8	2.14	2.30	0.16	0.98	0.16
9*	1.74	2.33	0.59	5.89	< 0.001
10*	1.33	2.75	1.42	10.18	< 0.001
11*	2.85	3.42	0.58	4.27	< 0.001
12*	2.14	2.52	0.38	3.18	0.001
13*	0.86	1.37	0.51	5.22	< 0.001
14*	1.70	2.00	0.30	2.80	0.003
15*	2.33	2.90	0.58	4.25	< 0.001
16*	2.73	3.21	0.49	3.56	< 0.001
Total score*	32.03	39.83	7.80	13.30	< 0.001

* Significant improvement at individual level of 0.05.

Performance by Topic

The following two-factor interaction model with blocking was used to analyse the pre-test and post-test scores by topic area:

Topic score = grand mean + prepost + topic + prepost*topic + block + error.

In the model, prepost is the pre-post effect, topic is a four-level "treatment" of the topic areas, and each student teacher constitutes one level of blocking. The interaction term prepost*topic allows comparisons between topics to be made in pre-test, in post-test, and across the two tests. The ANOVA results are shown in Table 2.

Table 2: *ANOVA table for topic scores*

Source	Df	MS	F	p-value
prepost	1	860.44	170.26	< 0.001

Topic	3	71.48	14.14	< 0.001
Block	112	15.22	3.01	< 0.001
prepost*topic	3	43.99	8.71	< 0.001
Error	784	5.05		
Total	903			

Pairwise comparisons of topic means based on Bonferroni 95% simultaneous confidence intervals were performed and the results are shown in Tables 3 and 4. Note that if we had used the usual (individual) 95% confidence intervals for each pairwise comparison, then the overall confidence for *all* comparisons is *less* than 95%. For example, if we use two 95% confidence intervals, then we can only claim an overall confidence of (at least) 90% for the two comparisons. Thus, by increasing the confidence of the individual intervals, the Bonferroni method provided an overall confidence of (at least) 95% for all the comparisons made in Tables 3 and 4. Pairwise comparisons of the pre-test and post-test means in Table 3 revealed significant improvements in all topics except *Fractions & Decimals*.

Table 3: *Pairwise comparisons of pre-test and post-test means by topic*

Topic Area	Pre-test mean	Post-test mean	Difference of means (post – pre)	Bonferroni 95% CI for difference of means	
				Lower	Upper
Whole Numbers	7.38	9.52	2.14*	1.21	3.08
Fractions and Decimals	8.98	9.81	0.82	-0.11	1.76
Geometry	8.06	11.02	2.96*	2.02	3.90
Measurement	7.61	9.49	1.88*	0.94	2.81

* Significant difference at overall level of 0.05.

Pairwise comparisons of the topic means in Table 4(a) indicated that in the pre-test, the *Fractions & Decimals* mean score was significantly higher than the *Whole Numbers* as well as the *Measurement* mean scores. However, these differences were no longer significant in the post-test. Instead, Table 4(b) indicated that in the post-test, the *Geometry* mean score was significantly higher than each of the other three mean scores.

Table 4: *Pairwise comparisons of topic means*

(a) Pre-test

Difference of means (Bonferroni 95% CI)	(a) Whole Numbers	(b) Fractions and Decimals	(c) Geometry
(b) Fractions and Decimals	1.60* (0.67, 2.54)		(b) – (c)
(c) Geometry	0.68 (-0.26, 1.62)	-0.93 (-1.86, 0.01)	
(d) Measurement	0.23 (-0.70, 1.17)	-1.37* (-2.31, -0.43)	-0.45 (-1.38, 0.49)

(b) Post-test

Difference of means (Bonferroni 95% CI)	(a) Whole Numbers	(b) Fractions and Decimals	(c) Geometry
(b) Fractions and Decimals	0.28 (-0.65, 1.22)		(b) – (c)
(c) Geometry	1.50* (0.56, 2.43)	1.21* (0.28, 2.15)	
(d) Measurement	-0.04 (-0.97, 0.90)	-0.32 (-1.26, 0.62)	-1.53* (-2.47, -0.59)

* Significant difference at overall level of 0.05.

Likely explanations for the findings given above are suggested by the items themselves and the courses which the student teachers have taken or both. In the two topics *Whole Numbers* and *Geometry*, the two "content" items (items 1 and 9) dealt with areas covered in the two mathematics content modules, namely, operations on classes of real numbers and properties of quadrilaterals which were taken by 34 student teachers out of 113. The post-test improved performances in these items contributed to the improvements in their respective topics.

As for the topic *Fractions & Decimals*, the student teachers' pre-test performance was at a relatively high level for item 7 and they did not improve much after the courses. Item 7 required the student teachers to explain the sequence in which they would use the 3 decimal numbers 0.2, 0.03 and 0.23 to teach the conversion of decimals into fractions. It was not an item with absolute answers but partial credit was given for reasonable justifications of different answers even if these were not clearly explained resulting in relatively higher scores. The anomalous item 6 as discussed above is also a contributing factor to the non-improvement of the *Fractions & Decimals* topic area. The marginal increase performance on the *Measurement* topic area relative to other topic areas in the post test was largely due to the weak performance of item 13 which required more science knowledge (the difference between mass and weight) than mathematics. This item was the second lowest scoring item in the pre-test and the lowest scoring item in the post-test.

Performance by MPCK Construct

For analysis of the pre-test and post-test scores by MPCK construct, a model similar to that for topic scores was used:

MPCK score = grand mean + prepost + mpck + prepost*mpck + block + error.

In the model, mpck is a four-level "treatment" of the MPCK constructs. The ANOVA results are shown in Table 5.

Table 5: *ANOVA table for MPCK scores*

Source	Df	MS	F	p-value
prepost	1	860.44	163.72	< 0.001
mpck	3	322.19	61.30	< 0.001
block	112	15.22	2.90	< 0.001
prepost*mpck	3	11.41	2.17	0.09
Error	784	5.26		
Total	903			

Pairwise comparisons of MPCK means based on Bonferroni 95% simultaneous confidence intervals are given in Tables 6 and 7. Pairwise comparisons in Table 6 revealed that the improvements were significant for all the four constructs. The mathematics pedagogy and content modules taken by the student teachers probably contributed to these improvements.

Table 6: *Pairwise comparisons of pre-test and post-test means by MPCK construct*

MPCK construct	Pre-test mean	Post-test mean	Difference of means (post – pre)	Bonferroni 95% CI for difference of means	
				Lower	Upper
(a) Teachers' own knowledge of mathematical structure and connections	6.11	8.47	2.36*	1.40	3.31
(b) Representations (multiple or alternative) of concepts	8.70	10.33	1.63*	0.67	2.58
(c) Cognitive demands of mathematical tasks on learners	8.88	11.20	2.32*	1.36	3.27
(d) Pupil difficulties and mis-conceptions and choice of actions	8.34	9.84	1.50*	0.55	2.46

* Significant difference at overall level of 0.05.

Pairwise comparisons of the MPCK means in Table 7(a) indicated that in the pre-test, the MPCK mean score for *Teachers' Own Knowledge of Mathematical Structures and Connections* was significantly lower than the other three mean scores. These differences continued to be significant in the post-test. In addition, Table 7(b) indicated that in the post-test, the mean score for *Pupil Difficulties and Misconceptions and Choice of Actions* was significantly lower than the score for *Cognitive demands of mathematical tasks on learners*.

Table 7: *Pairwise comparisons of MPCK means*

(a) Pre-test

Difference of means (Bonferroni 95% CI)	(a) Teachers' own knowledge of mathematical structure and connections	(b) Representations (multiple or alternative) of concepts	(c) Cognitive demands of mathematical tasks on learners
(b) Representations (multiple or alternative) of concepts for the purpose of explanations	2.59* (1.63, 3.54)		(b) – (c)
(c) Cognitive demands of mathematical tasks on learners	2.77* (1.81, 3.73)	0.18 (-0.77, 1.14)	
(d) Pupil difficulties and misconceptions and choice of actions	2.23* (1.27, 3.18)	-0.36 (-1.32, 0.59)	-0.54 (-1.50, 0.41)

(b) Post-test

Difference of means (Bonferroni 95% CI)	(a) Teachers' own knowledge of mathematical structure and connections	(b) Representations (multiple or alternative) of concepts	(c) Cognitive demands of mathematical tasks on learners
(b) Representations (multiple or alternative) of concepts	1.86* (0.90, 2.82)		(b) – (c)
(c) Cognitive demands of mathematical tasks on learners	2.73* (1.78, 3.69)	0.87 (-0.08, 1.83)	
(d) Pupil difficulties and misconceptions and choice of actions	1.37* (0.41, 2.33)	-0.49 (-1.45, 0.47)	-1.36* (-2.32, -0.41)

* Significant difference at overall level of 0.05.

The higher score for the *Cognitive demands of mathematical tasks on learners* MPCK construct in both the pre- and post- tests was primarily due to items 10 and 15. Item 10 had three parts to the item and gave the student teachers opportunity to provide reasonable answers in one or two parts even if they could not do so for the third part, thus explaining the higher score in this item. There was also significant improvement in this item between the pre- and post- test, possibly due to the concept of static and dynamic views of angles being covered in their mathematics methodology modules, thus resulting them having a good knowledge of static and dynamic views of angles. Item 15 provided two problems on the area of rectangle and required the participants to choose one of the problem to help pupils understand the formula for the area of rectangle. Most of them had chosen *Problem **B**: Sketch two rectangles each having an area of 12 cm*^{2} which they believe would deepen the pupils' understanding area of rectangle. Despite the strong performance on item 1 in the post-test, poorer performance in the *Teachers' Own Knowledge of Mathematical Structures and Connections* MPCK construct resulted due to the poor performance on items 5 and 13. Item 13 has already been discussed earlier in this paper. For item 5, student teachers found it difficult to reconcile different expressions of mathematics. Table 6 showed that the improvement in the four constructs in both the pre- and post- tests was significant.

Effect of Subject Knowledge Modules

For the PGDE 2005 cohort, 34 student teachers out of 113 were doing the two subject knowledge (SK) modules. The following two-factor interaction model with nested blocking was used to analyse the effect of the two SK modules on the pre-test and post-test total scores:

Total score = grand mean + prepost + sk + prepost*sk + block(sk) + error.

In the model, prepost is the pre-post effect, sk is a two-level "treatment" indicating whether a student teacher has done the SK modules, and each student teacher constitutes one level of blocking nested within sk. The interaction term prepost*sk allows comparisons between sk and non-sk student teachers to be made within each test and across the two tests. The ANOVA results are shown in Table 8.

Table 8: *ANOVA table for total scores*

Source	Df	MS	F	p-value
prepost	1	3319.11	176.37	< 0.001
sk	1	126.88	6.74	0.011
block(sk)	111	60.30	3.20	< 0.001
prepost*sk	1	90.94	4.83	0.03
Error	111	18.82		
Total	225			

Pairwise comparisons of pre-test and post-test means based on Bonferroni 95% simultaneous confidence intervals were performed and the results are shown in Table 9. The pre-post comparisons in Table 9 revealed significant improvements for both non-SK and SK student teachers. Furthermore, while there were no significant differences between non-SK and SK in the pre-test, student teachers doing SK performed significantly better than those not doing SK in the post-test.

Table 9: *Pairwise comparisons of pre-test and post-test means by SK (Bonferroni 95% confidence intervals for differences of means are given in parenthesis)*

SK modules	Pre-test mean	Post-test mean	Difference of means (Post – Pre)
Not doing SK	31.95	38.92	6.97* (5.12, 8.83)
Doing SK	32.20	41.94	9.74* (6.91, 12.57)
Difference of means (SK – Non-SK)	0.25 (-2.14, 2.64)	3.02* (0.63, 5.41)	

* Significant difference at overall level of 0.05.

Concluding Remarks

The findings and discussion show that there was significant improvement in the total scores on the MPCK instrument before and after the Postgraduate Diploma in Education (Primary) programme. It is heartening to note that the post-test scores as indicated by the MPCK instrument are reaching a desired level. The "weakest" MPCK construct was

Teachers' Own Knowledge of Mathematical Structure in both the pre- and post- test although there was significant improvement on this construct between the two tests. The National Institute of Education has sought to develop student teachers' content knowledge which is related to the primary mathematics curriculum through the content course described earlier. This 48-hour course was introduced to the programme in 2005 for selected student teachers and appears to have some effect in improving their knowledge of mathematics.

There was also indication that student teachers do not do well in timed tests which requires the ability to understand questions and answer them appropriately, clearly and concisely within a limited time. However, they tend to do well in the assignments and group projects of their methodology courses. There are assessment issues to be considered here for while it is argued that methodology course assessment should allow student teachers to have longer time to plan and to present their work, such longer assignments also mean that the student teachers need only focus on a few teaching topics and may have problems with other topics. Moreover, in teaching situations, a teacher will often need to rely on the knowledge he or she has to respond immediately with appropriate action and thus such modes of assessment may not gauge this aspect of a teacher's capability. Essentially, the assessment of a teacher's effectiveness is a complex issue and needs to be multi-dimensional. Teacher educators are constantly grappling with the issues and, while assessment as a field of study has been developing for some time, there is need for scholarly study in applying this field to the assessment of teacher education courses at universities.

The MPCK project is being extended to in-service teachers and student teachers of Diploma in Education programme (Lim-Teo, Chua, Cheang, & Yeo, 2006). In addition, report on the practices valued by Heads of Mathematics Departments in schools can be found in Lim-Teo, Ng & Chua (2006). Observable MPCK-in-action outcomes are also being studied through video-taping a small sample of teachers teaching mathematics. It is hoped that a clearer identification of what MPCK-in-action constructs are will enable teacher educators to better develop these in their student teachers.

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Acknowledgement

This paper is based on work from the research project Knowledge for Teaching Primary Mathematics (EP 1/03 MQ). The authors acknowledge the generous funding of the project from the Education Research Fund, Ministry of Education, Singapore.